TWO-DIMENSIONAL TRANSFORMATIONS
(Chapter 5 in *Computer Graphics*)

- Basic Philosophy
- Basic Transformations
- Matrix Representations and Homogeneous Coordinates
- Composite Transformations
- Other Transformations
- Transformation Commands
- Raster Methods for Transformations

Basic Philosophy

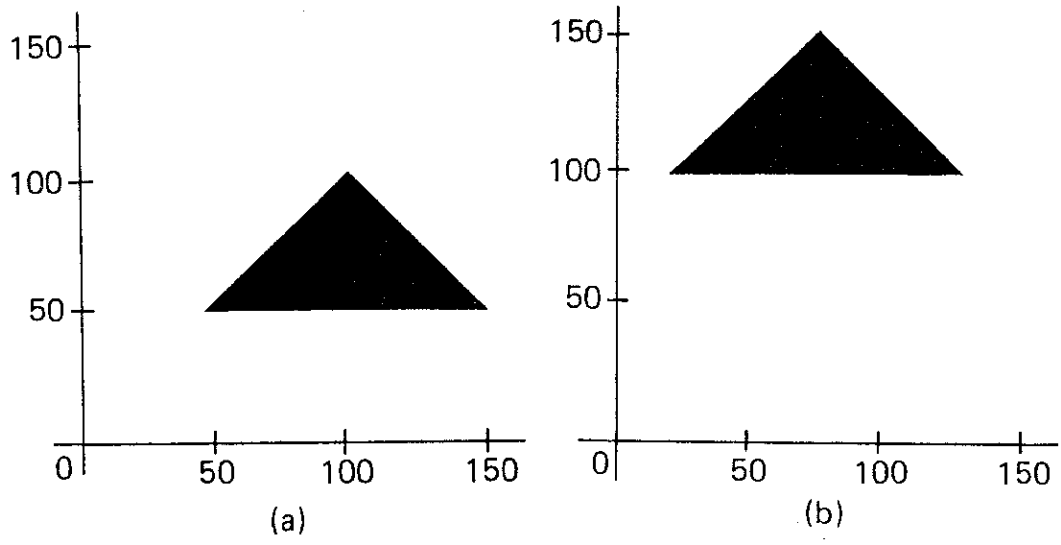
- objects are defined by sets of vertices
- objects are transformed by transforming their vertices
- straight lines stay straight

Basic Transformations

- translation (reposition)
- scaling (reduce or enlarge)
- rotation (reorient)

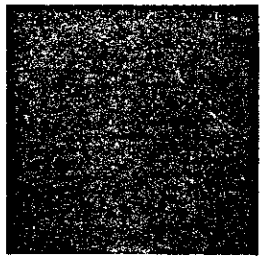
translation

- straight line movement from one position to another
- $x' = x + Tx$
  $y' = y + Ty$
- translate objects by adding the translation vector to the coordinates of each endpoint

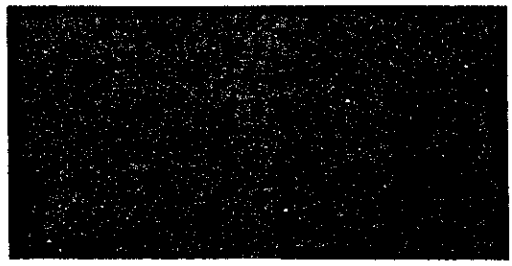


scaling

- altering the size of an object
- $x' = x \cdot Sx$
  $y' = y \cdot Sy$
- scale objects by multiplying the coordinates of each endpoint by the scaling factors



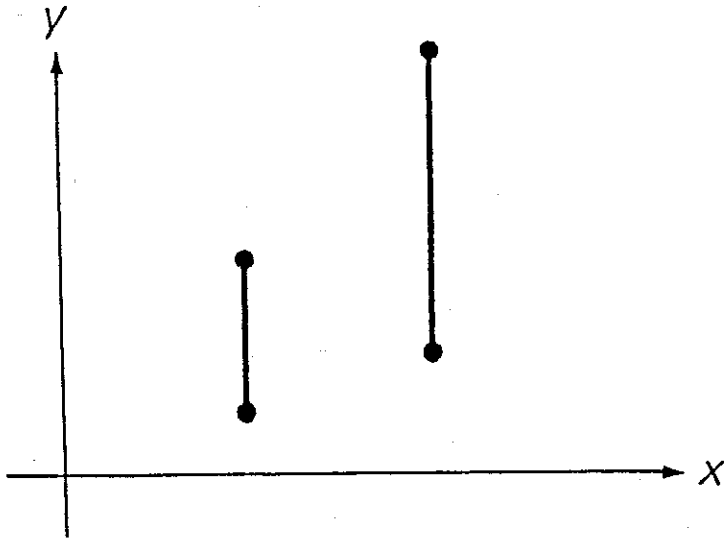
(a)



(b)

scaling, continued

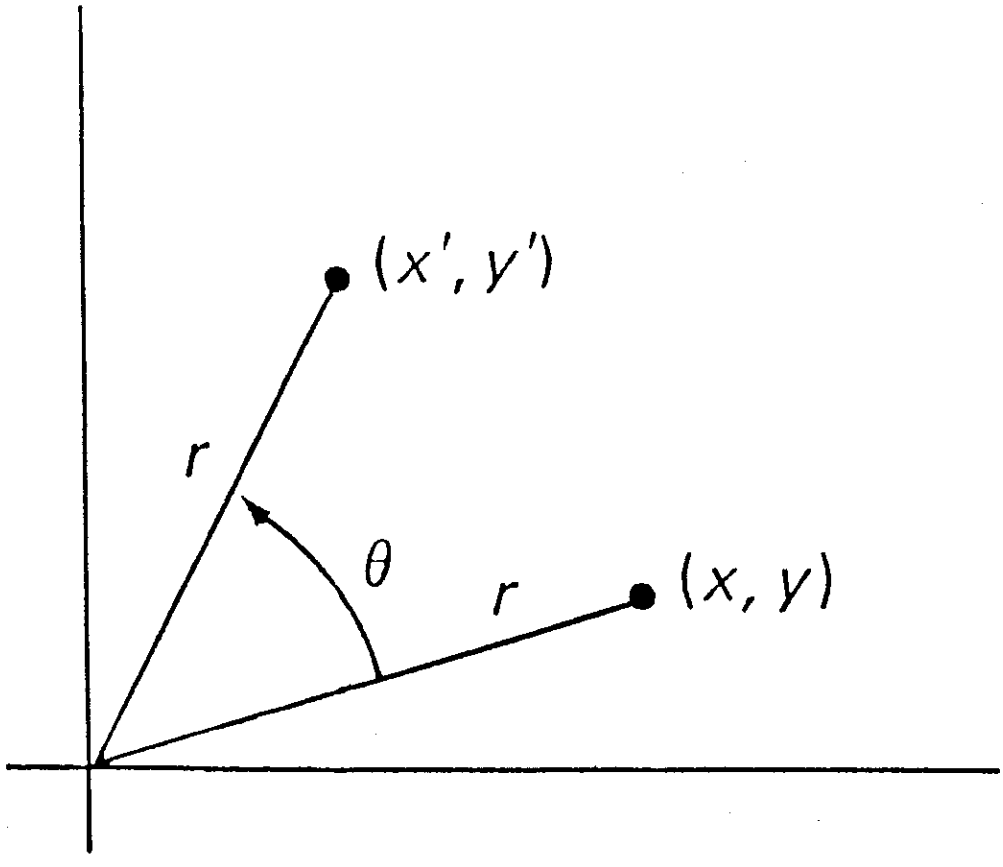
- lengths and distances from the origin are scaled



- one point (x_F, y_F) can remain fixed in position
- $x' = x_F + (x - x_F)Sx$
  $y' = y_F + (y - y_F)Sy$

rotation

- transformation along circular paths
  - x' = xcosΘ - ysinΘ
  - y' = ycosΘ + xsinΘ
- rotate objects by rotating each endpoint



rotation, continued

- rotation about an arbitrary pivot point (x_r, y_r)
  - $x' = x_r + (x - x_r)\cos\Theta - (y - y_r)\sin\Theta$
  - $y' = y_r + (y - y_r)\cos\Theta + (x - x_r)\sin\Theta$
- efficient rotation for small angles
  - $\cos\Theta = 1$
  - $\sin\Theta = \Theta$ in radians

Matrix Representations and Homogeneous Coordinates

- final coordinates are calculated directly from initial coordinates using matrix methods
- a homogeneous coordinates is added
  - (x,y) becomes [x y 1]

basic transformation matrices

- translation

$$[x'\ y'\ 1] = [x\ y\ 1]\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ Tx & Ty & 1 \end{bmatrix}$$

- scaling

$$[x'\ y'\ 1] = [x\ y\ 1]\begin{bmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- rotation

$$[x'\ y'\ 1] = [x\ y\ 1]\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

inverses of basic transformation matrices

- translation

$$[x'\ y'\ 1] = [x\ y\ 1]\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ & & 1 \end{bmatrix}$$

- scaling

$$[x'\ y'\ 1] = [x\ y\ 1]\begin{bmatrix} & 0 & 0 \\ 0 & & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- rotation

$$[x'\ y'\ 1] = [x\ y\ 1]\begin{bmatrix} & & 0 \\ & & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Composite Transformations

- a composite transformation matrix is the product of individual transformation matrices
- multiplying two matrices together is referred to as concatenation

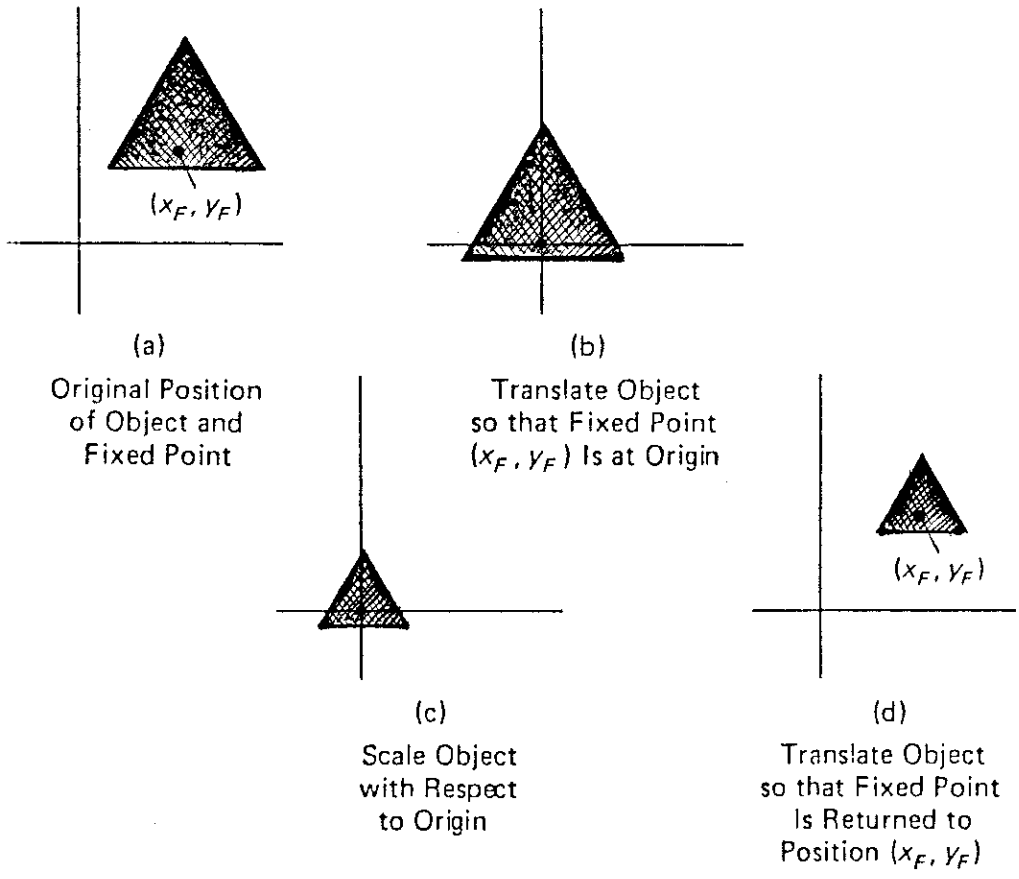
two successive translations

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ Tx_1 & Ty_1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ Tx_2 & Ty_2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ Tx_1 + Tx_2 & Ty_1 + Ty_2 & 1 \end{bmatrix}$$

scaling relative to a fixed point

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -x_F & -y_F & 1 \end{bmatrix} \cdot \begin{bmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ x_F & y_F & 1 \end{bmatrix} =$$

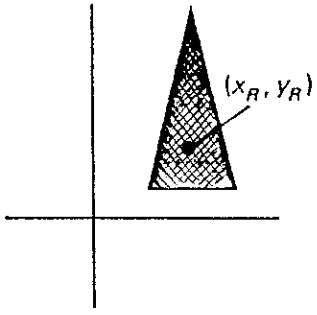
$$\begin{bmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ (1 - Sx)x_F & (1 - Sy)y_F & 1 \end{bmatrix}$$



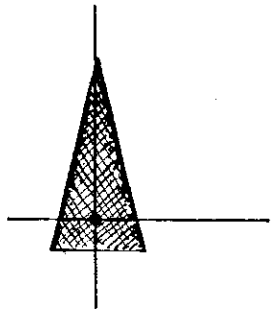
rotation about a pivot point

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -x_R & -y_R & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ x_R & y_R & 1 \end{bmatrix}$$

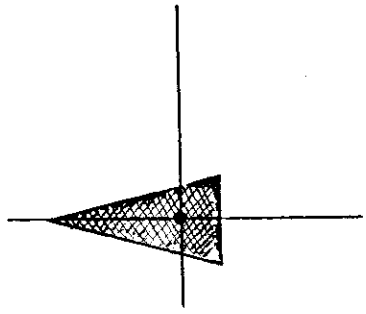
$$= \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ (1-\cos\theta)x_R + y_R \cdot \sin\theta & (1-\cos\theta)y_R - x_R \cdot \sin\theta & 1 \end{bmatrix}$$



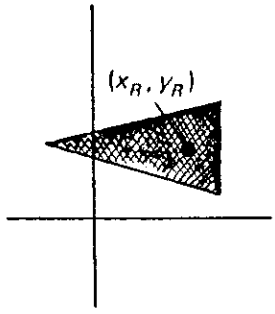
(a)
Original Position of Object and Pivot Point



(b)
Translation of Object so that Pivot Point (x_R, y_R) Is at Origin

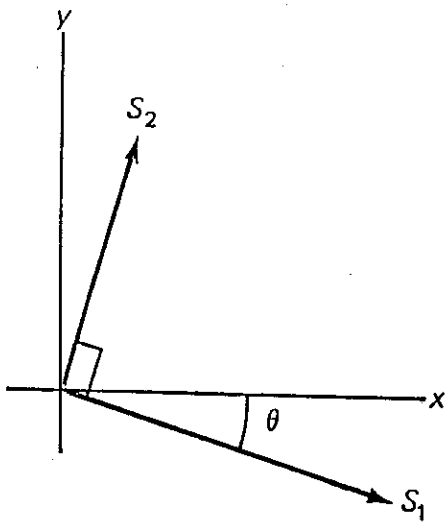


(c)
Rotation about Origin



(d)
Translation of Object so that the Pivot Point Is Returned to Position (x_R, y_R)

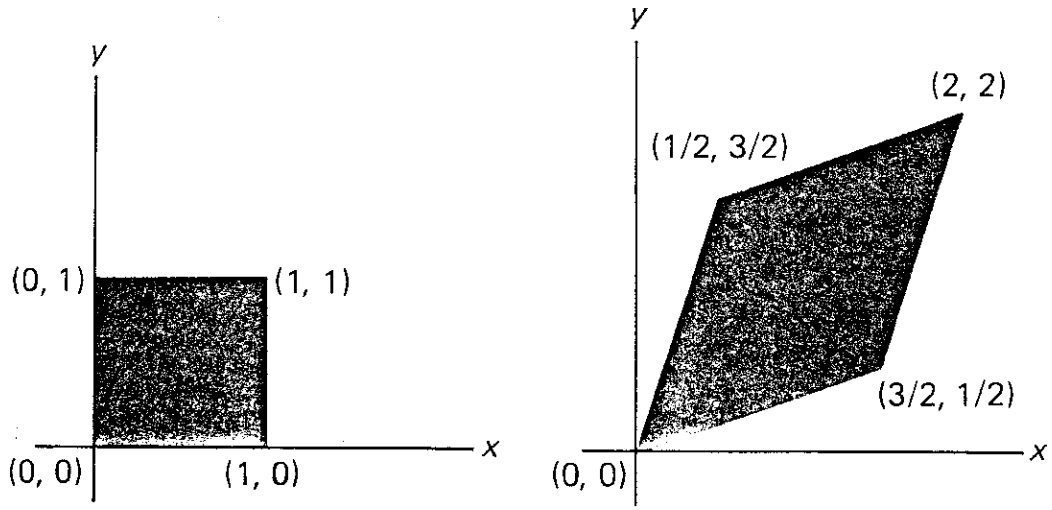
arbitrary scaling directions



- rotate so that the scaling axes coincide with the x and y axes
- scale
- rotate scaling axes back to their original positions

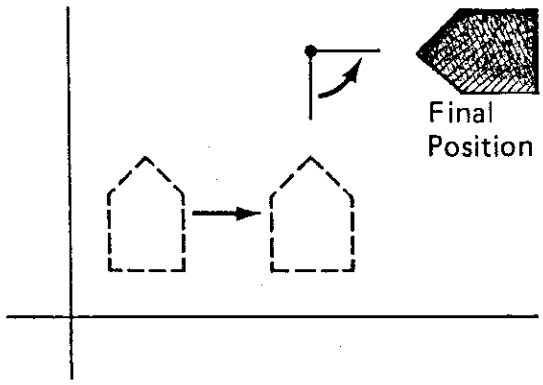
arbitrary scaling directions, continued

- example
  - $\theta = 45^{\circ}$
  - $S_1 = 1$
  - $S_2 = 2$

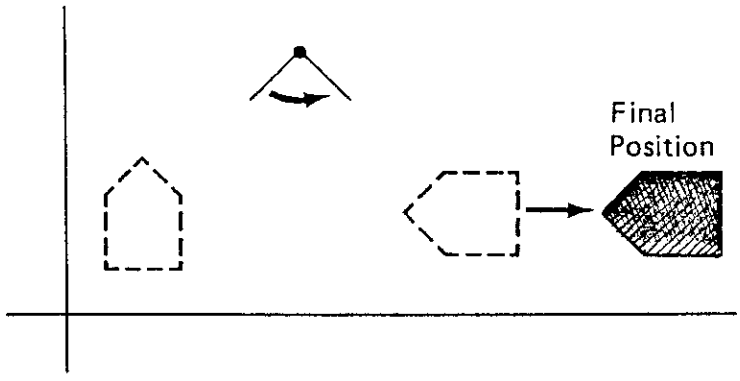


concatenation properties

- matrix multiplication is associative
- matrix multiplication is not commutative



(a)

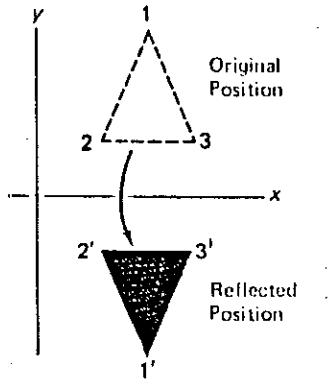


(b)

Other Transformations - reflection

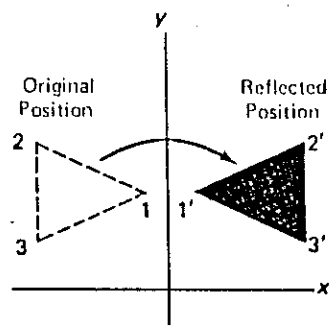
- produce a mirror image relative to an axis of reflection
- reflection about the x axis

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



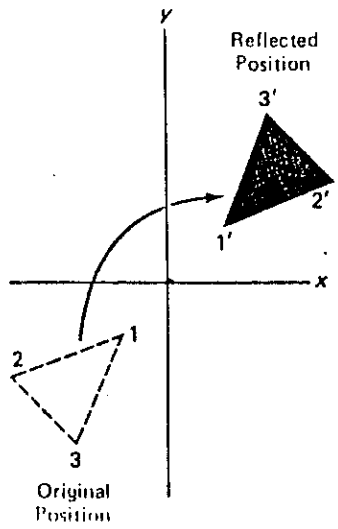
- reflection about the y axis

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



- reflection about the origin

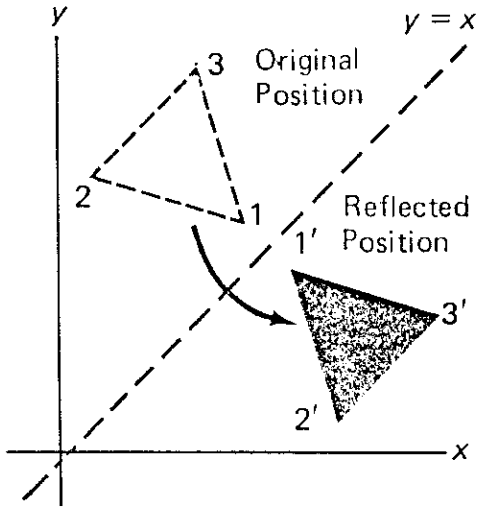
$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Other Transformations - reflection, continued

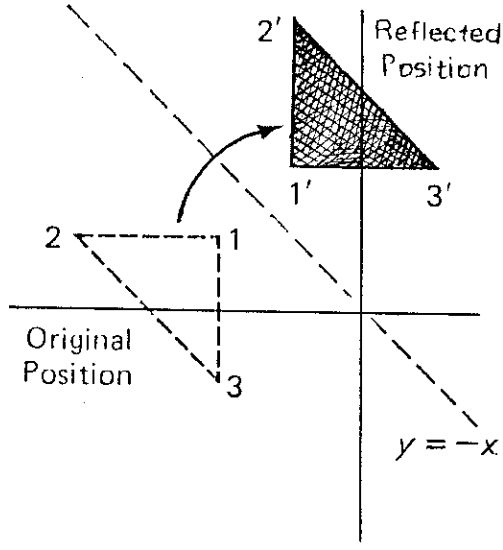
- reflection about the line y = x

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



- reflection about the line y = -x

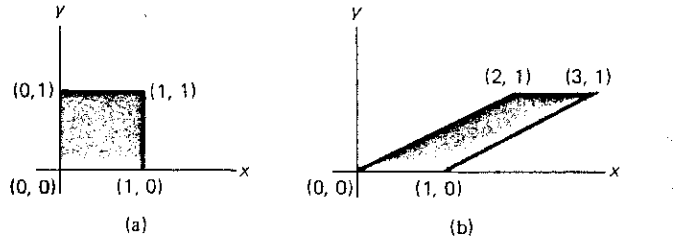
$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Other Transformations - shear

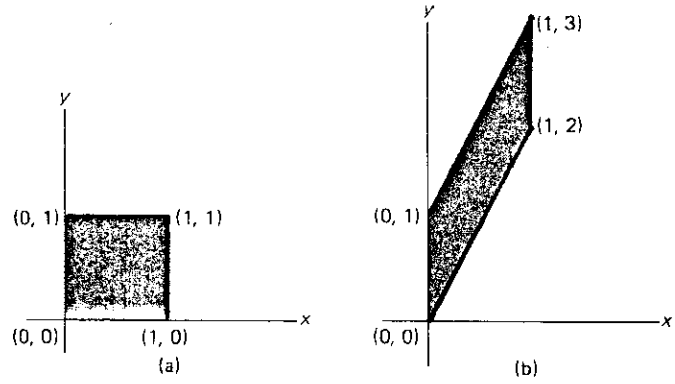
- produce a distortion
- x-direction shear

$$\begin{bmatrix} 1 & 0 & 0 \\ SHx & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



- y-direction shear

$$\begin{bmatrix} 1 & SHy & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Transformation Commands

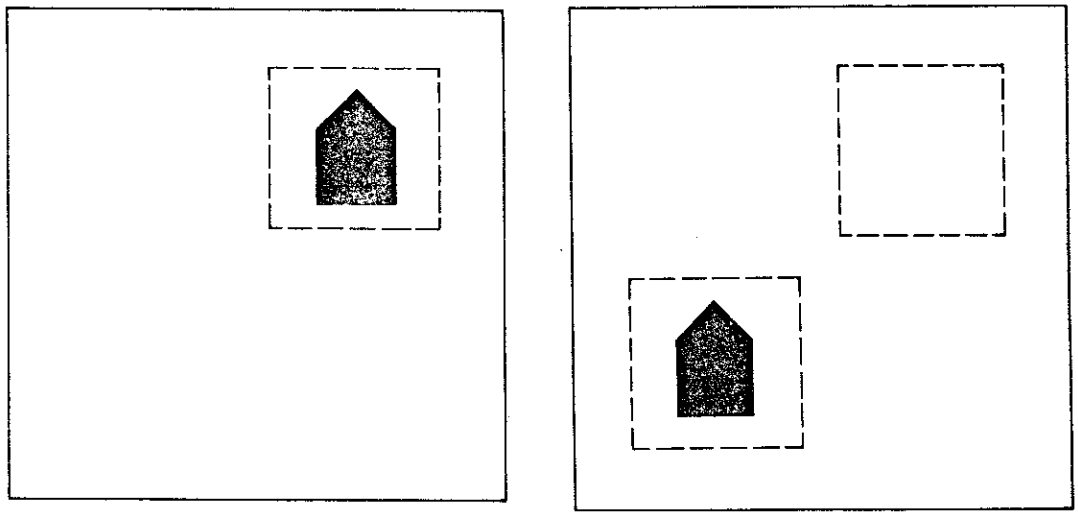
- sometimes separate commands for each transformation operation
- sometimes a composite transformation command
  - create_transformation_matrix(xf, yf, sx, sy, xr, yr, a, tx, ty, matrix)
    assumes order
    - scale
    - rotate
    - translate
  - could be used for a single transformation

Transformation Commands, continued

- concatenation
  - accumulate_transformation_matrix (matrix_in, xf, yf, sx, sy, xr, yr, a, tx, ty, matrix_out)
- selecting previously defined matrices
  - set_transformation (matrix)

Raster Methods for Transformations

- some simple transformations can be carried out by manipulating the frame buffer contents
- translation (bit block transfer or bit-blt)
  - copy a block from one area of the frame buffer to another area of the frame buffer
  - fill the old area with background color
  - begin with an overlapped corner



  - Boolean operations can be applied (exclusive-or is quite useful)
- rotation by 90^{O} increments
- scaling by integer multiples

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